Efficiency of a Small Magnetohydrodynamic Drive

Kiran Ghanekar

May 2024

1 Abstract

Magnetohydrodynamic drives (MHDs) are devices that accelerate a conductive liquid by applying perpendicular magnetic and electric fields across it. They are currently being studied for use in the military, primarily because they are silent. They are, however, rather inefficient. An MHD was constructed using two solenoids and two graphite electrodes, and its efficiency was measured. It reached its peak efficiency at an applied voltage of 7V, equivalent to an input power of 42.75W – at that point, it was $1.82\text{e}{-2}^{+4.08\text{e}-3}_{-2.82\text{e}-3}$. The measured efficiency distribution did not correspond well with the theoretical efficiency distribution – more precise testing with more ideal fluids will likely give results in greater alignment with theory.

2 Introduction

Magnetohydrodynamics is the study of electromagnetic effects on fluids – it is through this effect that Earth receives its magnetic field [1]. The Lorentz force is the force that a magnetic field exerts on a charged particle [2]. Magnetohydrodynamic drives (MHDs) are devices which use the Lorentz force to apply force to a fluid, without moving parts [1]. They have great potential in naval settings, and the US Defense Advances Research Projects Agency is currently attempting to develop an effective MHD system to apply to these settings. Their main advantages are that they are silent and have no moving parts, and are hence harder to detect [3]. Several MHD-powered ships were built at the end of the 20th century, but the projects failed because MHD drives are inefficient – it is difficult to create strong magnetic fields [1]. Further, the electrodes used tend to degrade quickly [3].

3 Background

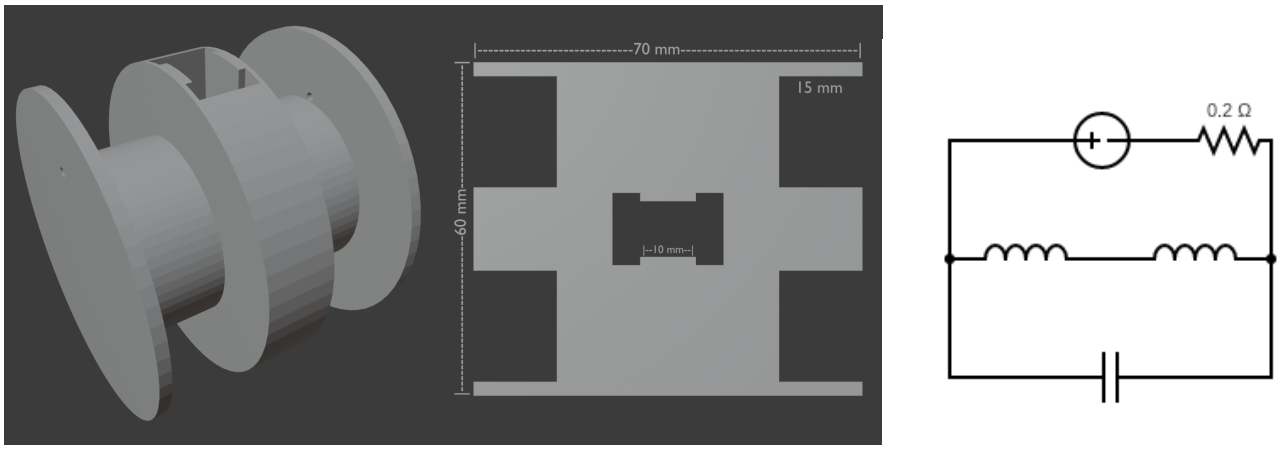
The Lorentz force on a volume is $f = \rho E + J \times B$, where f is the force density, ρ is the charge density, J is the current density, and B is the magnetic field. Assuming that Ohm's Law is true, $J = \sigma E$, where σ is the conductivity of the volume and E is the electric field across it [2]. When MHDs are used to accelerate aqueous solutions, $\rho = 0$ (the net charge of an aqueous solution is zero). Also, they are built such that B is perpendicular to J [1]. So the force on the solution is $f = \sigma EB$. Assuming a constant magnetic and electric field, using the laws of kinematics, the velocity at output is $v = \sqrt{\frac{2\sigma Bl}{\rho d}V}$, where ρ is the density of the solution, V is the charge across the electrodes, d is the width of the tube, and l is the length of the tube.

$P_{out} = vAp$ (power is velocity times cross-sectional area times pressure, since power is pressure times flow and flow is velocity times area) [4]. Since the power used by parallel circuits is additive, the input power is $P_{in} = \frac{V^2}{R} + \frac{\sigma V^2 A}{d} = V^2(\frac{1}{R} + \frac{\sigma A}{d})$, where R is the resistance of the solenoid circuit and A is the exposed area of the electrode. Therefore, assuming that everything but voltage remains constant, the input power is proportional to V^2 and the output power is proportional

to $\sqrt{V}$. So $P_{out} \propto P_{in}^{\frac{1}{4}}$. Further, $(E = \frac{P_{out}}{P_{in}}) \propto V^{-\frac{3}{2}}$. Please see appendix A for graphs of these functions.

4 Materials and Methods

The magnetohydrodynamic drive was constructed according to the following description. Using a 3D printed frame, two solenoids, with an inner diameter of 4 cm, an outer diameter of 7 cm, and a width of 2 cm, each with approximately 170 coils, were mounted on the top and bottom of a 1 by 1 cm square tube, 2.5 mm away from it. Please see appendix B for a characterization of the magnetic field. Two graphite electrodes were cut and fit into notches on either side of this tube, each one 10 cm long, 1.3 cm wide, and 0.5 cm thick.



(a) 3D view (b) Cross-sectional view (c) Circuit diagram

Figure 1: Specifications of the device

The end of the first solenoid was connected to the beginning of the second, so that they would be in series, with current flowing in the same direction through both. The graphite electrodes were connected in parallel with this circuit. A variable-voltage power supply, able to supply up to 100W of power, was constructed out of a 12V power supply and a buck converter, and attached. In the circuit diagram above, the 0.2Ω shunt resistor depicted is removed during actual testing, and the "capacitor" represents two electrodes and the liquid between them.

The device was submerged in an aqueous solution of 1% w/w NaOH, which has a conductivity of $5.32\frac{S}{m}$ [5]. This conductivity was chosen so that a wide range of sufficiently separate voltages could be tested – if it were too low, the full voltage of the power supply would not produce a significant effect; if it were too high, the power consumption would rise above what the power supply could provide even at low voltages.

All measurements were done consecutively, over the course of approximately an hour, so that temperature and pressure changes could be minimized. Measurements were taken between 2V and 10V. Using a 0.2Ω shunt resistor, the resistance of the circuit at each voltage to be tested was measured. Three trials were done for each voltage; before each trial, the tube was flushed with a syringe to clear the bubbles from hydrolysis. Then, dye was placed at the inlet with a syringe.

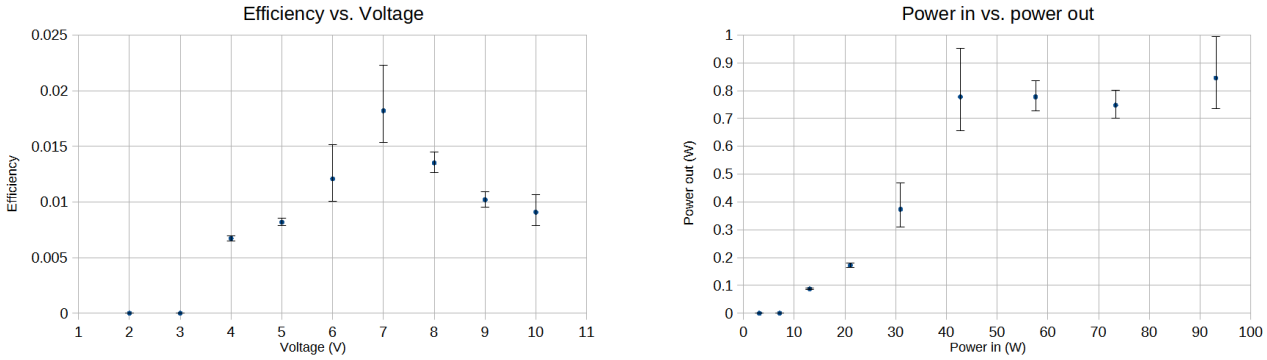
Input power was calculated from input voltage using Ohm's Law ($P = \frac{V^2}{R}$). The outlet, illuminated by a flashlight, was filmed at 240 fps with a slow-motion video camera, in order to determine the velocity of the fluid as it exited – the number of frames needed for the dye to

travel a certain easy-to-measure distance (27 mm) from the outlet was found and converted to velocity using $v = \frac{\text{distance} \times \text{frames per second}}{\text{frames}}$. Output power was calculated from the exit velocity using $P = vAp$; efficiency was found by dividing input and output power.

The mean and standard deviation in the number of frames was calculated for each voltage. The error bars in the graphs below come from propagating the error for each trial through the above equation. Please see appendix C for a table of the raw data.

5 Results

No motion could be detected at 2V and 3V. In figure 2(a), efficiency has a clear peak at an applied voltage of 7V, or an input power of 42.75W – at that point, it is $1.82\text{e}{-2}^{+4.08\text{e}-3}_{-2.82\text{e}-3}$. Starting at 4V, efficiency increases as 7V is approached, then decreases. In 2(b), power out increases with power in until 42.75W is reached, then plateaus and stays essentially constant.



(a) Scatter plot of efficiency vs voltage

(b) Scatter plot of P_{in} vs P_{out}

Figure 2: Plots of the results

6 Discussion and Conclusion

The plateau in 2(b) seems to explain the peak in 2(a) – the fluid velocity reaches a limit determined by some other factors, so increasing the input power only worsens the efficiency.

As discussed in the Background section, theoretically, 2(a) should look like $y = x^{-\frac{3}{2}}$. This is clearly not the case, at least in the first half – $y = x^{-\frac{3}{2}}$ has no maximum, and increases without bound as one approaches $x = 0$. 2(b) looks a bit more like $y = x^{\frac{1}{4}}$, especially in the second half, but whereas $y = x^{\frac{1}{4}}$ is always concave down, this plot is concave up for the first half, then essentially flat. These unexpected observations probably have to do with the fact that the theoretical calculations do not account for friction within the fluid – it is likely that some sort of static friction needs to be overcome for the fluid to start moving in the first place.

This experiment shows that it is possible to construct and test a small-scale MHD. Further experiments, using fluids with lower viscosity, and taking more precise readings at more voltages, may yield results that align more closely with theory. On the other hand, an experiment on this device that would vary the strength of the magnetic field (since the solenoid circuit consumes most of the power) would be useful to determine whether the peak efficiency can be increased.

7 References

[1] D. Cébron, S. Viroulet, J. Vidal, J.-P. Masson, and P. Viroulet, "Experimental and theoretical study of magnetohydrodynamic ship models," *PLOS ONE*, vol. 12, no. 6, Jun. 2017. DOI: 10.1371/journal.pone.0178599.

[2] D. J. Griffiths, *Introduction to electrodynamics*, 4th ed. Pearson Education, 2012. [Online]. Available: https://hansandcassady.org/David%20J.%20Griffiths-Introduction%20to%20Electrodynamics-Addison-Wesley%20(2012).pdf.

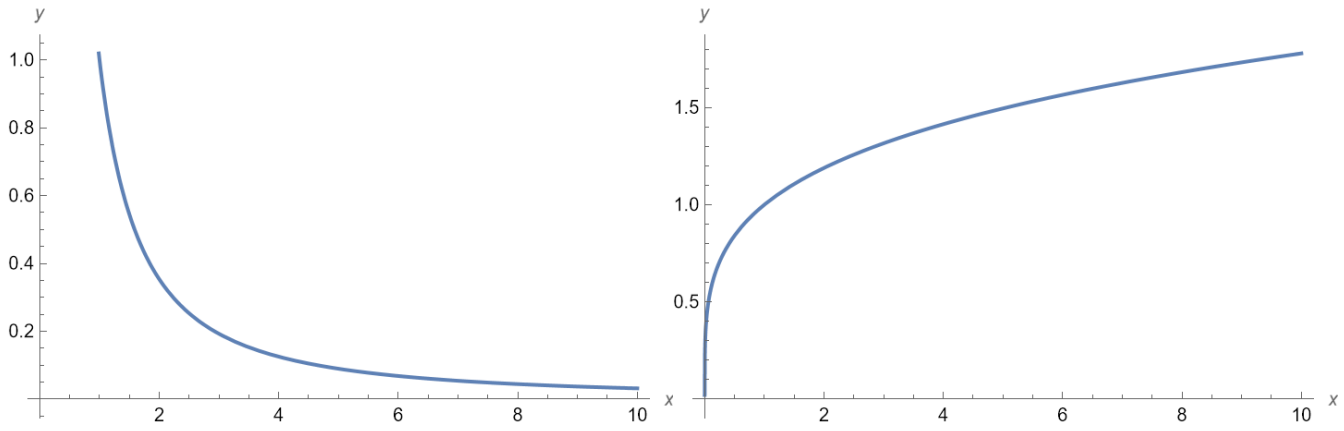
[3] A. Salerno-Garthwaite, *Darpa's silent MHD magnetic drives for replacing naval propellers*, May 2023. [Online]. Available: https://www.naval-technology.com/features/darpa-silent-mhd-magnetic-drives-for-replacing-naval-propellers/.

[4] *Fluid power fundamentals*. [Online]. Available: https://www.e4training.com/hyd_formula/power1.php.

[5] *Conductivity chart of liquids*. [Online]. Available: https://www.traskinstrumentation.com/pdf/app_notes/Conductivity_Chart_of_Liquids.pdf.

[6] *Finite element method magnetics*. [Online]. Available: https://www.femm.info/wiki/HomePage.

A Graphs of Theoretical Functions



(a) $y = x^{-\frac{3}{2}}$ (b) $y = x^{\frac{1}{4}}$

Figure 3: Graphs of the two ideal curves

B Magnetic Field Characterization

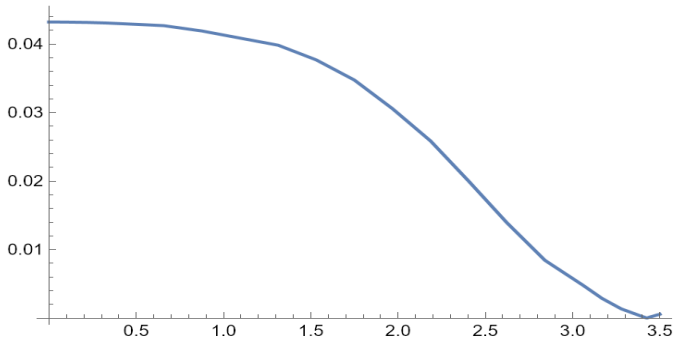


Figure 4: Ideal magnetic field strength at 10V, measured in T, as a function of the distance from the centre of the tube along its length (in cm), as simulated by FEMM [6]. The actual magnetic field was measured with a Hall effect sensor to be approximately 0.014T at the centre of the tube at this voltage, significantly smaller than the theoretical value.

C Raw Data

V	Ω	Frames	P_{in} (W)	Velocity ($\frac{m}{s}$)	P_{out} (W)	Efficiency
2	1.28	n/a	3.12	0	0	0
3	1.26	n/a	7.12	0	0	0
4	1.23	7.40e2 ± 2.65e1	1.30e1	$8.76e{-3}^{+3.25e-4}_{-3.02e-4}$	$8.76e{-2}^{+6.97e-3}_{-6.49e-3}$	$6.72e{-3}^{+2.49e-4}_{-2.32e-4}$
5	1.19	3.77e2 ± 1.53e1	2.10e1	$1.72e{-2}^{+7.27e-4}_{-6.70e-4}$	$1.72e{-1}^{+8.53e-3}_{-7.86e-3}$	$8.18e{-3}^{+3.46e-4}_{-3.19e-4}$
6	1.16	1.73e2 ± 3.51e1	3.09e1	$3.74e{-2}^{+9.50e-3}_{-6.30e-3}$	$3.74e{-1}^{+1.52e-2}_{-1.00e-2}$	$1.21e{-2}^{+3.07e-3}_{-2.04e-3}$
7	1.15	8.33e1 ± 1.53e1	4.28e1	$7.78e{-2}^{+1.75e-2}_{-1.20e-2}$	$7.78e{-1}^{+2.23e-2}_{-1.54e-2}$	$1.82e{-2}^{+4.08e-3}_{-2.82e-3}$
8	1.11	8.33e1 ± 5.77	5.76e1	$7.78e{-2}^{+5.79e-3}_{-5.04e-3}$	$7.78e{-1}^{+1.45e-2}_{-1.26e-2}$	$1.35e{-2}^{+1.01e-3}_{-8.75e-4}$
9	1.10	8.67e1 ± 5.77	7.33e1	$7.48e{-2}^{+5.34e-3}_{-4.67e-3}$	$7.48e{-1}^{+1.09e-2}_{-9.56e-3}$	$1.02e{-2}^{+7.28e-4}_{-6.37e-4}$
10	1.07	7.67e1 ± 1.15e1	9.31e1	$8.45e{-2}^{+1.50e-2}_{-1.11e-2}$	$8.45e{-1}^{+1.07e-2}_{-7.89e-3}$	$9.08e{-3}^{+1.61e-3}_{-1.19e-3}$